CBCS SCHEME

USN | | | | | | | | | |

18MCM/MAR/IAE/MTR11

First Semester M.Tech. Degree Examination, Dec.2019/Jan.2020
Applied Mathematics

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Define : (i) Absolute errors (ii) Relative errors (iii) Percentage errors
If $R = \frac{4x^2y^3}{z^4}$ and errors in x, y, z be 0.001 then show that the maximum relative error at $x = y = z = 1$ is 0.009. **(07 Marks)**

b. By using bisection method, obtain an approximate root of the equation $\sin x = \frac{1}{x}$ that lies between x = 1 and x = 1.5 (measured in radians). Carry out six iterations. **(06 Marks)**

c. Find the root of the equation $xe^x - \cos x = 0$ by the method of false position correct to three decimal places. **(07 Marks)**

OR

2 a. Explain Newton-Raphson method to find an approximate roof of the equation f(x) = 0. Using this method obtain the approximate value of $(17)^{1/3}$ starting with $x_0 = 2$. **(07 Marks)**

b. Using Secant method, obtain an approximate root of the equation $x \log_{10} x = 1.2$. **(06 Marks)**

c. Find a positive root of the equation $f(x) = x^2 - 2x - 3 = 0$ using fixed point iteration method correct to four decimal places. **(07 Marks)**

Module-2

3 a. Perform three iterations of Muller's method to find the smallest positive root of the equation $f(x) = x^3 - 13x - 12 = 0$ with $x_0 = 4.5$, $x_1 = 5.5$ and $x_2 = 5$. **(10 Marks)**

b. A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of the time t (in seconds).

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and the angular acceleration of the rod, when t = 0.6 seconds. **(10 Marks)**

OR

4 a. Using Romberg's integration method, evaluate
$$\int_0^{1/2} \frac{x}{\sin x}\,dx$$, correct to four decimal places with h = 0.25, 0.125, 0.0625. **(10 Marks)**

b. Evaluate $\int_0^2 e^{-x^2}\,dx$ by taking seven ordinates using (i) Trapezoidal rule (ii) Simpson's $3/8^{th}$ rule (iii) Weddle's rule. **(10 Marks)**

Module-3

5 a. Solve the given system of equations using Cramer's rule : $x - y - 2z = 3$, $2x + y + z = 5$, $4x - y - 2z = 11$. **(05 Marks)**

b. Using Cholesky's triangularization method, obtain the solution of the system of equations:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$ **(07 Marks)**

c. Find the inverse of the co-efficient matrix and hence solve the system of equations $x_1 + x_2 + x_3 = 1$; $4x_1 + 3x_2 - x_3 = 6$; $3x_1 + 5x_2 + 3x_3 = 4$ using Gauss - Jordan elimination. **(08 Marks)**

OR

6 a. Using partition method, find the inverse of the matrix

$$A = \begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix}.$$ Hence, solve the system of equations $Ax = B$ where $B = \begin{bmatrix} -10 \\ 8 \\ 7 \\ 5 \end{bmatrix}$ **(10 Marks)**

b. Solve the system of equations $x_1 + x_2 + x_3 + x_4 = 2$; $2x_1 - x_2 + 2x_3 - x_4 = -5$; $3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$; $x_1 - 2x_2 - 3x_3 + 2x_4 = 5$, using Gauss – Elimination method. **(10 Marks)**

Module-4

7 a. Using Jacobi's method, find the all the eigen values and the corresponding eigen vectors of the matrix $A = \begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$. **(08 Marks)**

b. Using Given's method, transform the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ to tridiagonal form. **(05 Marks)**

c. Find the dominant eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ using Power method, taking the initial eigen vector as $[1 \quad 1 \quad 1]^T$ in five iterations. **(07 Marks)**

OR

8 a. Find all the eigen values of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix}$ using the Rutishauser method. Perform six iterations. **(07 Marks)**

b. Using Householder's method, reduce the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ into a tridiagonal matrix. **(06 Marks)**

c. Using Inverse Power method, find the eigen value of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ nearest to 3, taking $X = [1\ 1\ 1]^T$ as the initial eigen vector. **(07 Marks)**

Module-5

9 a. Consider two functions T and S, such that $T : \mathbb{R}^2 \to \mathbb{R}^2$ and $S : \mathbb{R}^2 \to \mathbb{R}^2$ are defined by

$T\left[\begin{pmatrix} x \\ y \end{pmatrix}\right] = \begin{bmatrix} 2x+y \\ 0 \end{bmatrix}$ and $S\left[\begin{pmatrix} x \\ y \end{pmatrix}\right] = \begin{bmatrix} x+y \\ xy \end{bmatrix}$

Determine whether T, S and the composite SoT are linear transforms. **(07 Marks)**

b. Using Gram-Schmidt orthogonalization, find an orthogonal basis for the span of the vectors $w_1, w_2 \in \mathbb{R}^3$ if $w_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$; $w_2 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$. **(06 Marks)**

c. Find the least squares solution of the inconsistent system Ax = B for $A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$ **(07 Marks)**

OR

10 a. Let $A = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$; $u = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$; $v = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$; $w = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$ and $T: \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $T(x) = Ax$

(i) Find T(u)

(ii) Find x in $\mathbb{R}^2$ such that T(x) = v

(iii) Verify whether 'w' is in the range of transformation. **(10 Marks)**

b. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$; Also for every $x, y \in \mathbb{R}^2$; $\langle x, y \rangle = x^T A y$ defines an inner product space on $\mathbb{R}^n$ then

(i) Prove that the unit vectors $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ are not orthogonal in the inner product space $\mathbb{R}^2$.

(ii) Find an orthogonal basis $\{v_1, v_2\}$ of $\mathbb{R}^2$ from the basis $\{e_1, e_2\}$ using Gram-Schmidt orthogonalization process. **(10 Marks)**

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